Starter Questions

Find the equation of the normal to the curve $y = 2x^2 + x - 10$ which is perpendicular to the line $6x - 2y + 3 = 0$

[5 marks]

Find the equation of the normal to the curve $y = 2x^2 + x - 10$ which is perpendicular to the line $6x - 2y + 3 = 0$

[5 marks]

$6x - 2y + 3 = 0 \Rightarrow y = 3x + \frac{3}{2}$	**M1**	Gradient of straight line

F Exponentials and logarithms

F6

Use logarithmic graphs to estimate parameters in relationships of the form $y = ax^n$ and $y = kb^x$, given data for x and y.

Assessed at AS and A-level

Teaching guidance

Students should be able to:

- reduce a non-linear relationship to linear form
- plot a graph from given data, drawing a line of best fit by eye and using it to calculate the gradient and intercept to estimate for unknown constants.

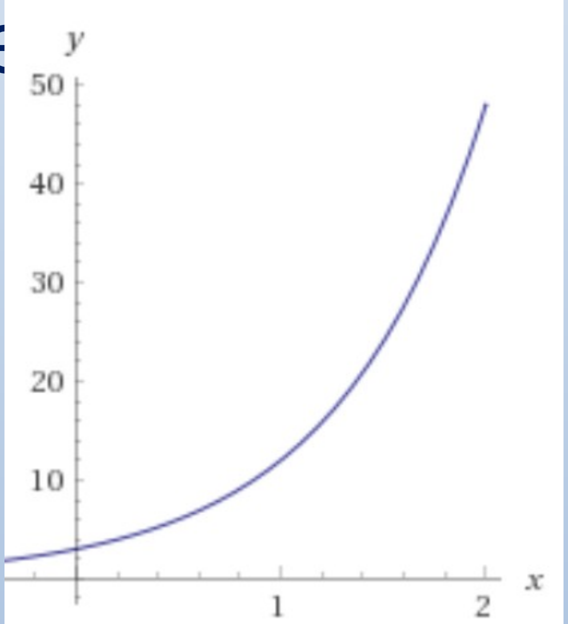
Note: this is an essential skill in A-level sciences and there is an ideal opportunity here to link to real data: power laws for relationships of the form $y = ax^n$ and exponential laws for those of the form $y = kb^x$

5.4 Curve Fitting

Logarithmic Graphs in Linear Form
Exponential Relationships

Consider the function
This graph gives a curve and, as before,
as increases, increases quite

e.g.



5.4 Curve Fitting

So if $$y = k b^x$$

then $$\log y = \log\left(k b^x\right)$$

$$\log y = \log k + \log b^x$$

$$\log y = x \log b + \log k$$

$$\mathbf{\log y = ¿¿}$$

5.4 Curve Fitting

$$\log y = ¿ ¿$$

As before, this can be written in linear form and plotted to give a straight line where

$$\boxed{Y ¿ mX + ¿ C ¿}$$

$$Y = \log y$$

$$Gradient\ m = \log b$$

$$Intercept\ C = \log k$$

5.4 Curve Fitting

Example 1

A patch of algae grows so that its area is $y\,\text{cm}^2$ after x days.

a Use the data in this table to find the relationship between x and y in the form $y = kb^x$

	2	4	6	8
	4.9	7.4	12.6	18.7

$$y = k\,b^x$$

$$\log y = \log\left(k\,b^x\right)$$

$$\log y = \log k + \log b^x$$

$$\log y = x\log b + \log k$$

$$\log y = ¿¿$$

We only need to log this time!

5.4 Curve Fitting

Example 1

A patch of algae grows so that its area is y cm^2 after x days.

a Use the data in this table to find the relationship between x and y in the form $y = kb^x$

	2	4	6	8
	4.9	7.4	12.6	18.7
	0.69	0.87	1.10	1.27

Now plot against log …

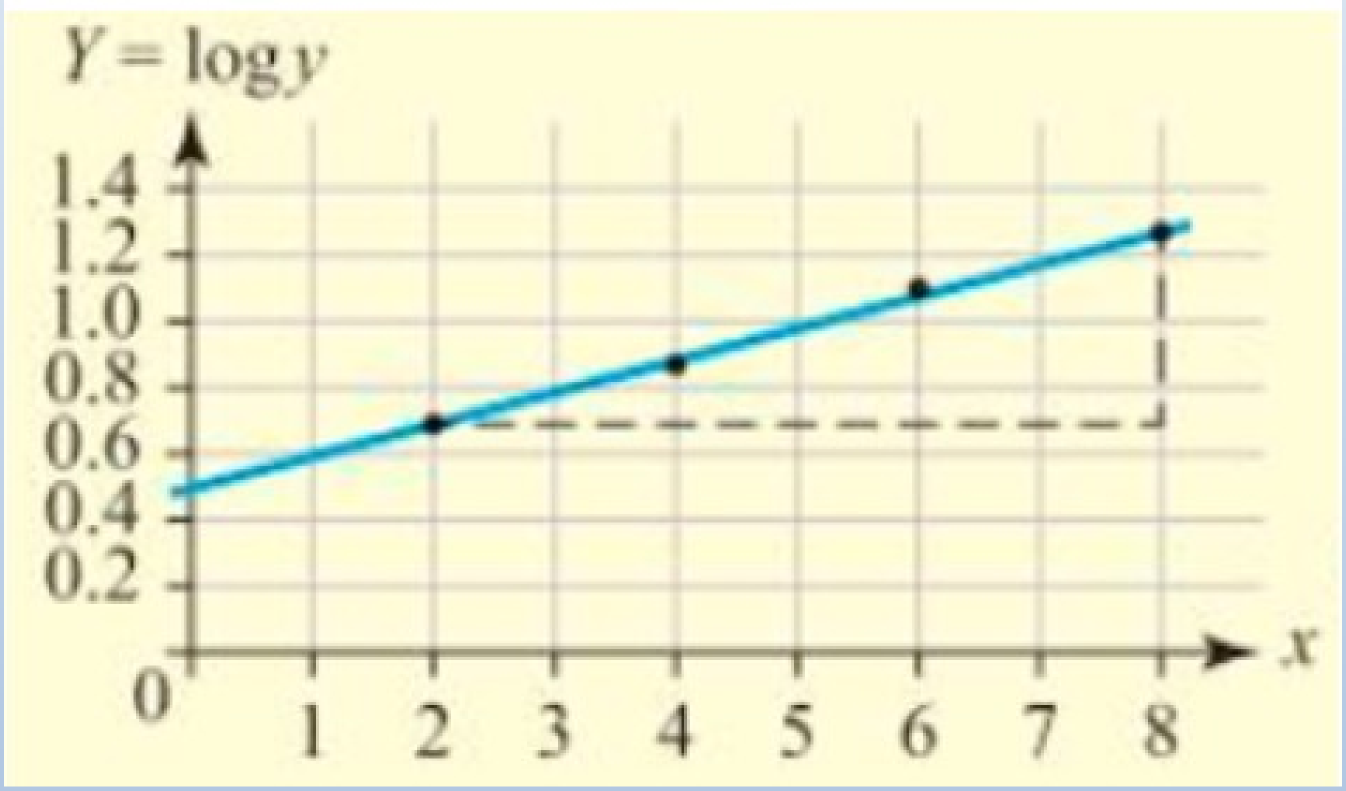
5.4 Curve Fitting

	2	4	6	8
	0.69	0.87	1.10	1.27

$$\log y = ¿¿$$

From the graph:

Vertical intercept:



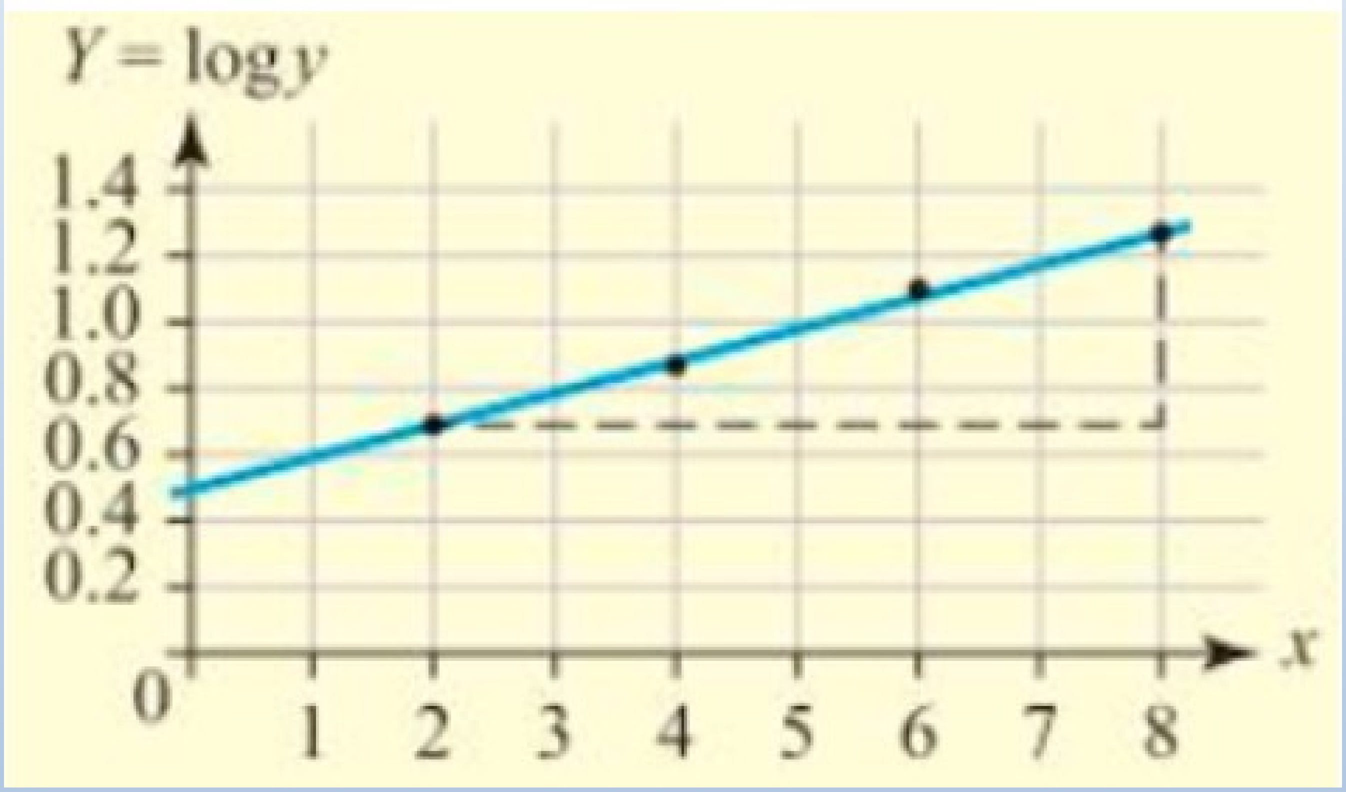
5.4 Curve Fitting

	2	4	6	8
	0.69	0.87	1.10	1.27

From the graph:

Gradient:

$$\log y = ¿¿$$



5.4 Curve Fitting

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	0.69	0.87	1.10	1.27

5.4 Curve Fitting

Example 1

A patch of algae grows so that its area is $y\,\text{cm}^2$ after x days.

a Use the data in this table to find the relationship between x and y in the form $y = kb^x$

b What was the initial area of algae and what would you expect the area to be after 10 days?

Initial area, i.e.

10 days, i.e.

5.4 Curve Fitting

Example 2a

A young couple decide to build a house but the project is delayed due to ongoing poor weather and contractor issues. Each month, the couple make an estimate of the total cost of the house. The table shows the estimated cost, £y thousand, of the project t months after the project was started.

Months after the project was started, t	1	2	3	4	5	6
Cost, £y thousand	280	289	332	371	408	458

The relationship between y and t is modelled by $y = ab^t$, where a and b are constants.

a. Show that $y = ab^t$ may be written as

$$\log_{10} y = \log_{10} a + t \log_{10} b$$

$$y = ab^t \Rightarrow \log_{10} y = \log_{10}(ab^t)$$
$$\Rightarrow \log_{10} y = \log_{10} a + \log_{10} b^t$$
$$\Rightarrow \log_{10} y = \log_{10} a + t \log_{10} b$$

5.4 Curve Fitting

Example 2b

A young couple decide to build a house but the project is delayed due to ongoing poor weather and contractor issues. Each month, the couple make an estimate of the total cost of the house. The table shows the estimated cost, £y thousand, of the project t months after the project was started.

Months after the project was started, t	1	2	3	4	5	6
Cost, £y thousand	280	289	332	371	408	458

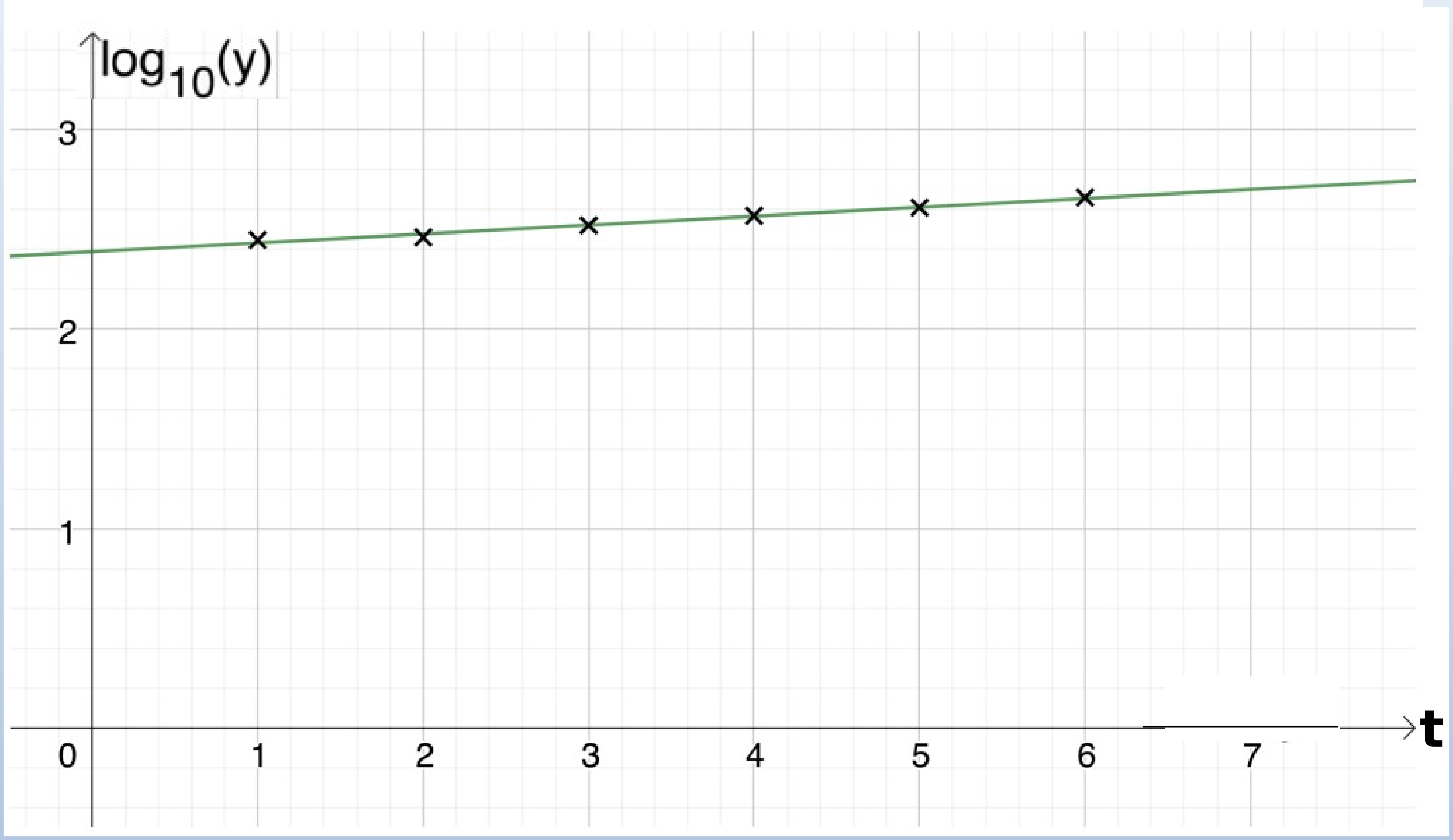
The relationship between y and t is modelled by $y = ab^t$, where a and b are constants.

b. Complete the table of values below and plot $\log_{10} y$ against t, drawing by eye a line of best fit.

t	1	2	3	4	5	6
y	280	289	332	371	408	458
$\log_{10} y$	2.44716	2.4609	2.52114	2.56937	2.61066	2.66087

5.4 Curve Fitting

Example 2b



5.4 Curve Fitting

Example 2c

A young couple decide to build a house but the project is delayed due to ongoing poor weather and contractor issues. Each month, the couple make an estimate of the total cost of the house. The table shows the estimated cost, £y thousand, of the project t months after the project was started.

Months after the project was started, t	1	2	3	4	5	6
Cost, £y thousand	280	289	332	371	408	458

The relationship between y and t is modelled by $y = ab^t$, where a and b are constants.

c. Use your graph to find the equation for y in terms of t.

Gradient ≈ 0.0447
Intercept ≈ 2.39

$$P = 245 \times 1.11^t$$

5.4 Curve Fitting

Example 2d

A young couple decide to build a house but the project is delayed due to ongoing poor weather and contractor issues. Each month, the couple make an estimate of the total cost of the house. The table shows the estimated cost, £y thousand, of the project t months after the project was started.

Months after the project was started, t	1	2	3	4	5	6
Cost, £y thousand	280	289	332	371	408	458

The relationship between y and t is modelled by $y = ab^t$, where a and b are constants.

d. Find the value of t given by this model when the estimated cost is £600 thousand. Give your answer rounded to 1 decimal place.

$$600 = 245 \times 1.11^t$$

$$t = \log_{1.11}\left(\frac{600}{245}\right) = 8.6 \text{ months to 1dp}$$

Complete the remaining three